Coefficients of Friction

Kinetic and Static Frictional Forces

Background

Have you ever noticed that pushing a heavy box over a smooth surface (like a tiled floor) requires less effort than pushing it over a rough surface (like carpet or concrete)?

This effect is due to the different frictional forces between the box and the smooth surface and the box and the rough surface. To quantify or compare the amount of friction between any two surfaces, scientists employ the **coefficients of friction**.

There are two types of friction: **static** and **kinetic**. Hence, there are two coefficients of friction: the **coefficient of static friction** and the **coefficient of kinetic friction**.

Static Frictional Force & Maximum Coefficient of Static Frictional Force

When you first apply a relatively small force to the heavy box, the box may not move at all. The force that opposes the force you apply is called **static friction**.

As you continue to apply an increasing force $\vec{F}$, the static frictional force opposes the applied force up to some maximum amount, initially preventing you from moving the box; it is in the opposite direction of $\vec{F}$ (Figure 1).

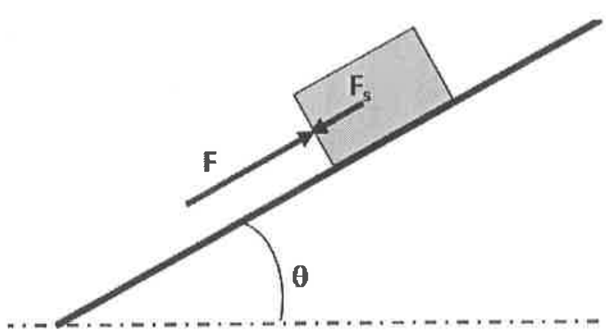

Figure 1: Force Applied to an Object and Static Force in Opposition

Equations relevant to the the **maximum static frictional force** are:

$$\vec{F}_s = \mu_s \vec{F}_N$$

$$\mu_s = \frac{\left\|\vec{F}_s\right\|}{\left\|\vec{F}_N\right\|}$$

where $\vec{F}_s$ is the maximum static frictional force in Newtons, μ_S is the dimensionless **coefficient of maximum static friction**, and $\vec{F}_N$ is the **normal force** (in Figure 2, the normal force is denoted by $\vec{F}_{norm}$).

What is the **normal force**? It is the force that is *opposing* the force exerted by gravity and is perpendicular to the plane along which the object is being pushed[1].

If the plane is inclined at an angle θ with the horizontal, then

$$\left\|\vec{F}_N\right\| = m \cdot |g| \cdot \cos(\theta).$$

In Figure 2, we see the relationship between $\vec{F}_N$, the incline of the plane, the mass of the object, and the force due to gravity $\vec{F} = m \cdot \vec{g}$. That is,

$$\vec{F}_N = \vec{F} \cdot \cos(\theta)$$

$$\vec{F}_N = m \cdot \vec{g} \cdot \cos(\theta)$$

[1] The word **normal** is used to describe a vector that is perpendicular to a plane or to a tangent line to a curve. The word **orthogonal** is used to describe two vectors that are perpendicular to each other.

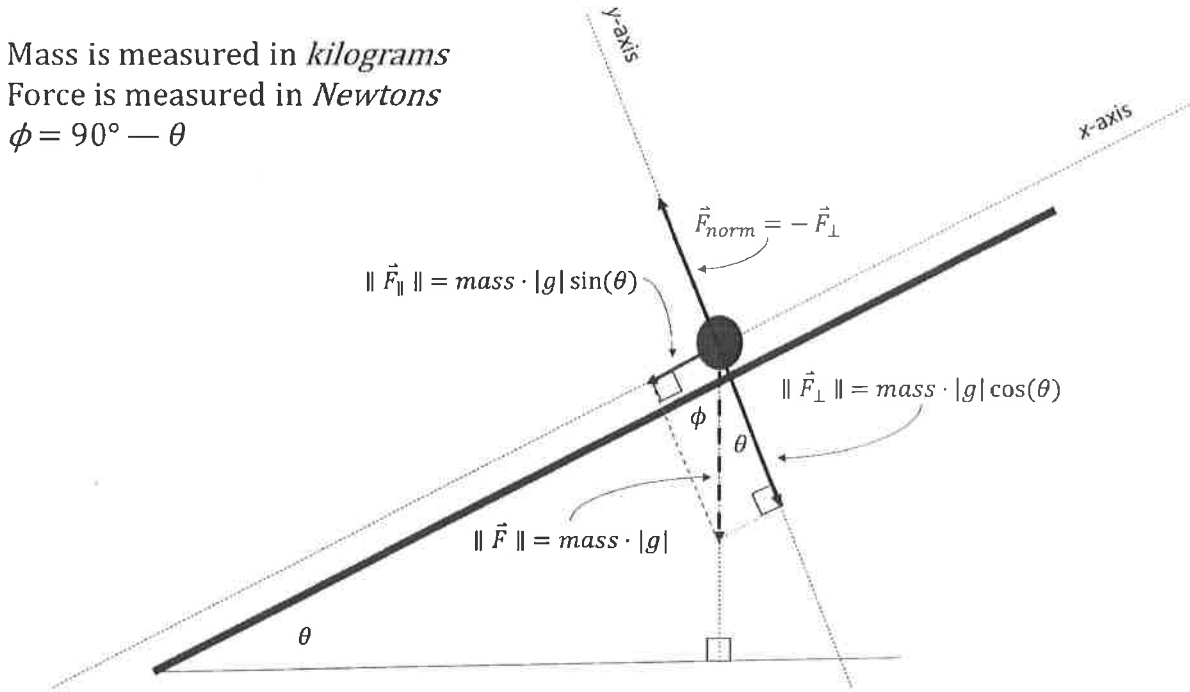

Figure 2: Mass on an Inclined Plane with Normal Force Shown

Thus, the coefficient of maximum static friction can be found via the formula

$$\left\|\vec{F}_s\right\| = \mu_s \vec{F} \cdot \cos(\theta) = \mu_s \cdot m \cdot |g| \cdot \cos(\theta) \text{ Newtons}$$

where $|g| = 9.81$ meters per second squared and *m* is the mass of the object in kilograms.

Again, static frictional force opposes the force you apply when trying to move the box along the surface. The magnitude of this force will depend on the material that the box is made from and upon the material the surface is made of. In other words, the coefficient of static friction depends on the materials that are making contact, just before enough force is applied to initiate motion.

For example, the coefficient of maximum static force is 1.6 for an object made of copper against a dry surface also made of copper. The coefficient is about 0.08 if the copper surface is lubricated.

The coefficient is a constant, without dimensions or units, and is independent of the mass of the object. However, the static frictional **force** itself is directly proportional to the normal force exerted on the object. If the normal force changes, so does the static frictional **force**, but not the **maximum coefficient** of static frictional force.

Calculation Method in Practice

Suppose the incline is adjusted so that the angle of inclination is sufficient to cause the object to begin to slide down the surface. Some practitioners use this angle, called the **ramp angle**, to estimate the maximum coefficient of static friction. Since this angle corresponds to the instance when $\vec{F}_s = \vec{F}_{||}$ (Figure 2), we have

$$\mu_s \cdot m \cdot |g| \cdot \cos(\theta) = m \cdot |g| \cdot \sin(\theta)$$

$$\mu_s = \frac{m \cdot |g| \cdot \sin(\theta)}{m \cdot |g| \cdot \cos(\theta)} = \tan(\theta).$$

In other words, tilt the plane just enough so that the object begins to slide, measure the angle of inclination, and the tangent of that angle corresponds to μ_s.

Kinetic Frictional Force & the Coefficient of Kinetic Frictional Force

If you continue to increase the force applied to an object, in an effort to move it, then eventually you will exert enough force to exceed the maximum static friction and the box will begin to move[2]. Once the box is moving, there will be **kinetic frictional force** opposing your exerted force. Kinetic frictional force slows the object down as you push, requiring you to exert even more force to move the object!

Important equations related to **kinetic frictional force** are given by

$$\vec{F}_k = \mu_k \vec{F}_N$$

$$\mu_k = \frac{\|\vec{F}_k\|}{\|\vec{F}_N\|}$$

where μ_k is the **coefficient of kinetic friction** and $\vec{F}_N$ is the same **normal force** as before.

Kinetic frictional force opposes the force that you exert on the object and its magnitude is again proportional to the **normal force**. The **coefficient of kinetic frictional force** is a constant and is unitless (dimensionless).

[2] For our purposes, we will assume constant velocity, at least for a short period of time after motion begins, in order to simplify our models. A constant velocity equates to an acceleration of 0 meters per second squared; thus, we can say that we will assume no acceleration from this point forward.

If you apply no force in opposition to the kinetic frictional force, then the object will be free to slide down the inclined plane. The acceleration will depend on the surface material, the object's material, the mass of the object, and the angle of inclination.

Here's an example you may have encountered. Have you ever complained about other peoples' driving abilities when it's raining? Accidents seem to happen more when it rains, especially when driving downhill or accelerating when a traffic light turns green. Maybe drivers could learn a bit from the fact that the coefficient of kinetic friction ranges from 0.75 - 0.85 for rubber tires on a *dry* asphalt road, but it ranges from 0.5 - 0.7 for rubber tires on a *wet* asphalt road. Thus, the magnitude of kinetic frictional force is less when the road is wet, and so it is quite easy to accelerate too much and quickly overtake μ_k, especially in a small, light vehicle.

Force Required to Put the Object in Motion

Remember: **static** means no motion; the net force is $\vec{0}$ Newtons ($\vec{F} = \vec{F}_s$ in Figure 1) and our system is in equilibrium. On the other hand, **kinetic** means motion; the net force is non-zero.

How much force must you apply before the object moves? That will depend on the mass of the object, the material that the object is made from, the type of surface along which you are attempting to move the object, and the incline of the plane along which you are pushing the object.

Over time, and as the force you apply increases, the static frictional force will match and oppose your exerted force until the static frictional force reaches a maximum magnitude. If you continue to push with enough force to overcome this maximum, then the object will move at a relatively constant velocity as long as the force you apply has a greater magnitude than the **kinetic frictional force** (Figure 3).

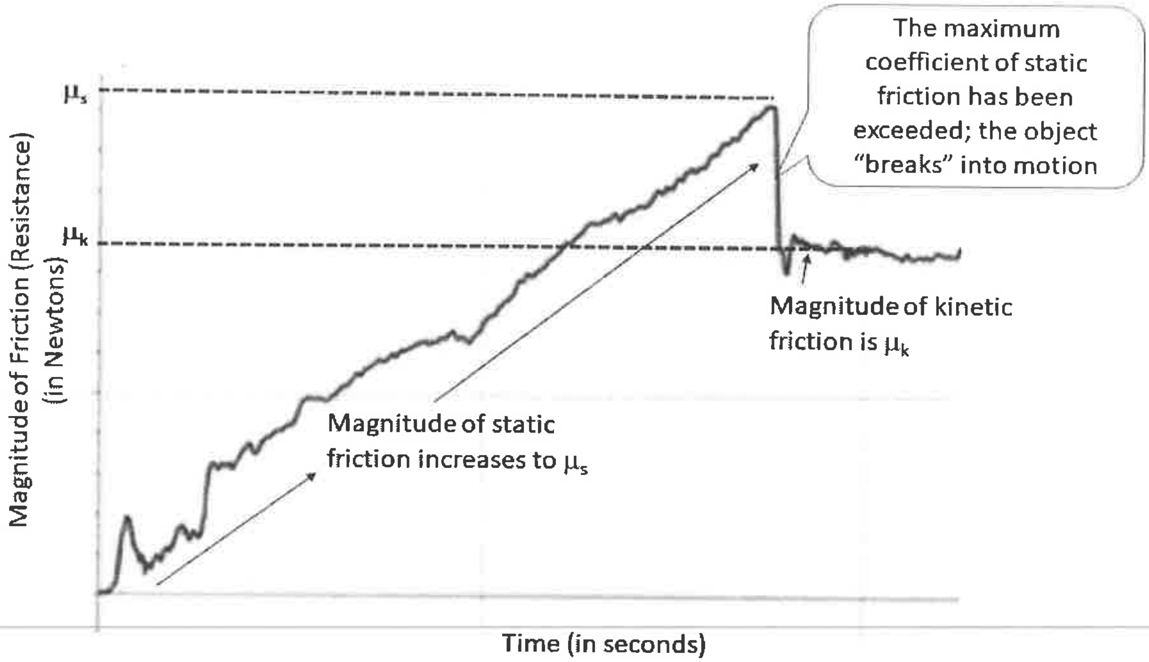

Figure 3: Plot of the Magnitude of Friction versus Time

In Figure 3, the object is motionless in the beginning, though static friction is increasing and opposing the increasing force exerted.

Suddenly, static friction reaches its maximum—you have overcome μ_s. This is sometimes called the *break* or *slip* in friction and appears as a sharp turn in Figure 3.

If you are exerting an increasing force of $\vec{F}(t)$, the net force is initially $\vec{F}(t) - \mu_s \vec{F}_N$; until this net force exceeds $\vec{0}$, the object is not going to move. The acceleration due to your exerted force is equal and in opposition to the acceleration due to static frictional force.

Until the **coefficient of static friction** reaches its maximum μ_s, $\vec{F}(t) < \mu_s \vec{F}_N$. At the break, $\vec{F}(t) = \mu_s \vec{F}_N$. And just for a moment, after the break, $\vec{F}(t) > \mu_s \vec{F}_N$. Static friction has been overcome and the acceleration due to your exerted force is greater than the acceleration due to the static frictional force. In other words, there is a brief moment where the net acceleration is positive. Once this happens, we have to focus on the other frictional force, as static frictional force is no longer relevant.

Because the object is now *in motion*; the system is *kinetic*, not *static*. There is opposition to this motion, too, because of the resistance between the object and the surface along which it is being pushed.

Kinetic friction is opposing your exerted force $\vec{F}(t)$, and you must continue to exert that same force to keep the object in motion at the same velocity achieved at the break.

We sometimes call the normal force $\vec{F}_N$ the *contact force*. Under the right conditions, with the compatible materials and appropriate temperatures, there might be some molecular bonding occurring, too.

Laboratory Objectives

After completing this experiment, you will determine the coefficients of **static friction** and **kinetic friction** (jointly called the **coefficients of friction**) between an object pulled across the surface and the surface itself.

- Establish an understanding of **static frictional force** and **kinetic frictional force** and the difference(s) between the two;
- Use a graphical method to determine an experimental value for each coefficient of friction.

Important Note

In this experiment, we will *pull* an object along a surface, rather than push it. The object will be pulled across a horizontal surface, rather than an inclined surface. Because the surface will be horizontal, $\theta = 0$ degrees and $\cos(\theta) = 1$ for all formulas given above.

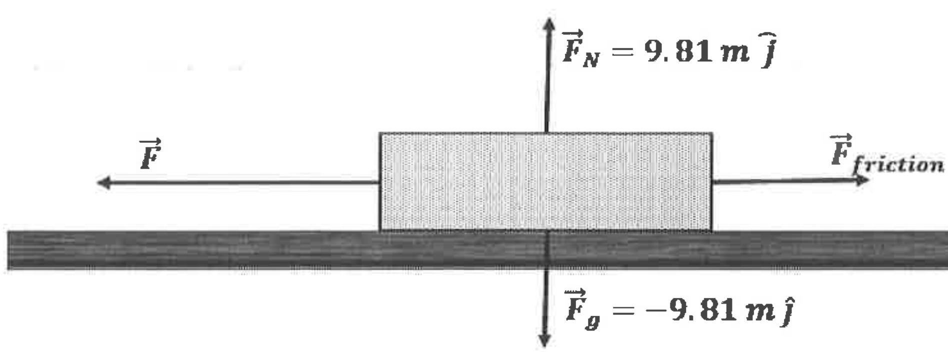

Figure 4: Diagram Demonstrating Forces in Action During Horizontal Motion

Thus, $\left\|\vec{F}_s\right\| = \mu_s \cdot m \cdot g$ and $\left\|\vec{F}_k\right\| = \mu_k \cdot m \cdot g$ for our system, and the forces of friction will be in the opposite direction from the direction in which we pull the carts.

Materials and Equipment

- GLX Data Collection System
- Laptop
- USB cord to connect GLX to laptop
- PASCO Capstone Software
- Force sensor with hook attachment
- Friction Carts x 3 (one each with plastic, felt, and cork bases)
- Friction board
- 0.250 kg cart masses (4)
- Balance
- String (30 cm x 3)
- PHYS210_Lab6.cap file
- Optional: table clamp to prevent friction board from moving
- Optional: USB drive for saving images and files

Safety

Normal laboratory safety features should be followed while completing this experiment.

Be careful not to drop the masses or the equipment off of the table. The force sensor is expensive and masses can injure toes and feet. The carts are plastic and can break if dropped.

If you choose to use a table clamp (C-clamp) to hold the board stable, make sure not to tighten it too much or it will damage the board.

Procedure

Please pay careful attention to the steps given below and listen to your instructor when she explains the steps.

As usual, some steps were completed for you before you entered the lab room.

Occasionally, you will require your instructor's permission to continue on to the next step; these occasions are indicated with the symbol .

Set Up

1. Connect the force sensor to the first port on the data collection device.
2. Connect the data collection device to the laptop with the USB cable.
3. Open Capstone and access the force sensor settings; make sure to check the box to "change the sign" as we will be pulling the plastic carts which is normally considered as a negative force. Since we want the magnitudes of observed forces to be recorded as positive values.
4. Set the force sensor to record at a rate of 250 Hz.
5. Set the recording to conditions to stop after 4 seconds have elapsed.
6. Attach a string to each friction cart by tying a small knot to secure the string to the cart. Make a loop at the other end of the string for attaching the string to the hook on the force sensor.

Part I: Cart with Felt Base

I.1. Tare the unattached force sensor by pressing the Zero button.

I.2. Using the balance, find the mass of the cart with the felt base, in grams, with the string attached and one mass loaded into the cart. Convert the mass to kilograms and record this value in the "Friction Data" table (second tab) in the appropriate location.

I.3. The magnitude of the normal force for the loaded cart will be automatically calculated in the data table in Capstone.

I.4. Place the cart on the friction board near one end.

I.5. Loop the string that is attached to the cart over the hook on the force sensor.

I.6. Make sure that you are holding the force sensor so that the string is horizontal.

I.7. While viewing the force vs time graph in the first tab, "Data Collection", in Capstone, press the record button.

I.8. Pull the force sensor **gently** so that the cart moves at a constant velocity and continue pulling until the cart nears the other end of the board or until recording stops. Do **not** move the force sensor up or down; keep it as level as possible with the cart.

I.9. Center the graph of the data.

I.10. Click the "Apply Smoothing" button  and increase the smoothing rate to about 21.

I.11. Locate the maximum force in the force vs time graph; this should correspond to the maximum coefficient of static friction. Click the data point on the graph and choose to display the point's coordinates. Move the label if you wish.

I.12. Right click on the coordinate label and select the tools option. In the options for the vertical coordinate, change the settings to show 3 significant figures. Close the tools menu.

I.13. Record the maximum force as the magnitude F_s in the "Friction Data" table.

I.14. Return to the "Data Collection" tab. Click the data highlighting button  and resize/reposition the rectangle to highlight all data to the right of the "slip" in the graph (Figure 5).

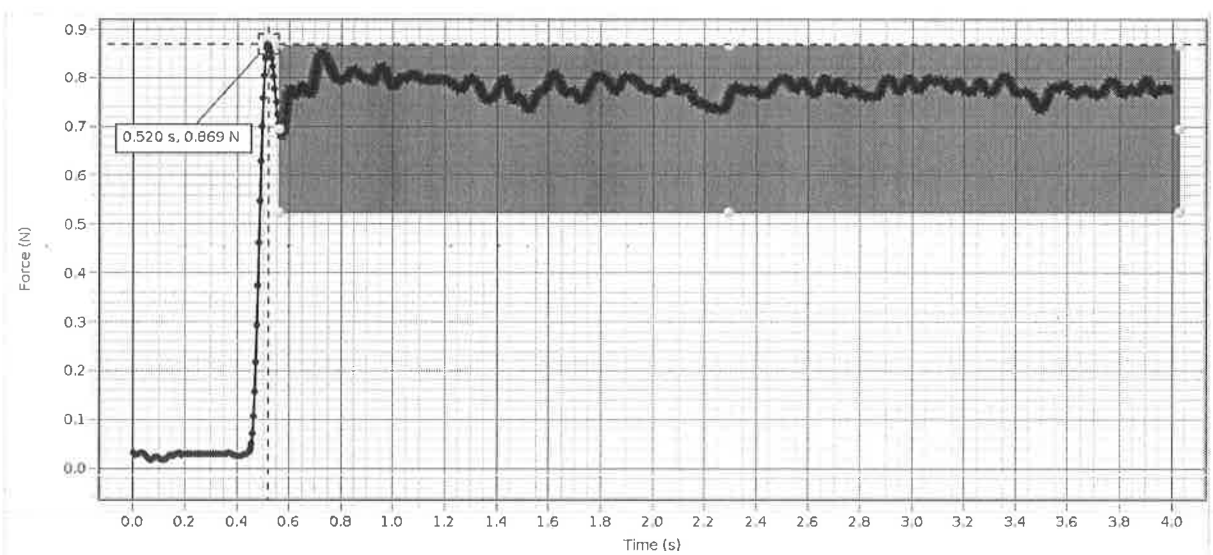

Figure 1: Force vs Time

Figure 5: Force vs Time Graph

I.15. Click on the down arrow next to the button on the graph and make sure that only *mean* is selected. If necessary, click the button again to display the mean of the highlighted data. It will appear on the far left of the graph. This value is an *estimate* for the magnitude F_k; record its value in the appropriate location in the data table.

I.16. Capstone is set up to automatically calculate the coefficients of friction after you enter the values for mass, F_s, and F_k.

Table 1: Force and Coefficient Data (Felt on Table)

F_N=[Accel due to gravity (m/s²)]*[Mass (kg)]

	Trial	Total Mass	Normal Force	Static Force	Coefficient of Static Friction	Kinetic Force	Coefficient of Kinetic Friction
	System	Mass (kg)	F_N (N)	F_s (N)	mu_s	F_k (N)	mu_k
1	Cart + 0.250 kg	0.339	5.78	0.869	0.197	0.780	0.165
2	Cart + 0.500 kg	0.589	8.22	1.620	0.205	1.360	0.169

Figure 6: Portion of the Friction Data Table in PHYS210_Lab6.cap

I.17. Ask your instructor to verify that your graph and data entries are reasonable.

I.18. Repeat steps I.1 - 1.17 after adding another 0.250 kg mass to the cart. Repeat again for a third and a fourth added mass. In other words, repeat the procedure with the cart loaded with 0.500 kg, 0.750 kg, and 1.00 kg of mass.

I.19. Look at the graphs of Static Force vs Normal Force and Kinetic Force vs Normal Force in the "Friction vs Normal" tab. If necessary, click the model button to add a direct variation model to the graph. To do this, choose the "Proportion" option to create a model of the form $F_x = A \cdot F_N$. Based on our preliminary work, the coefficients of friction should correspond to the respective slopes of these linear models.

I.20. Change the titles for the graphs, if necessary, to ensure that the material on the base of the cart is part of the title. Make copies of these graphs for inclusion in your lab report.

I.21. Enter the data values observed and those computed in the "Summary Table" in Capstone for future reference.

I.22. Compare the slopes found by the models to the mean of the coefficients of friction found in the data table by computing the percent difference:

$$\frac{|mean\ coefficient - slope|}{\left|\frac{mean\ coefficient + slope}{2}\right|} \times 100\% = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \%$$

Part II: Cart with Plastic Base

II.1. Make copies of any graphs, tables, or other elements of Capstone that you will wish to include in your lab report before proceeding.

II.2. Repeat the procedure in Part I for the cart with the plastic base. Record your percent difference below:

$$\frac{|mean\ coefficient - slope|}{\left|\frac{mean\ coefficient + slope}{2}\right|} \times 100\% = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \%$$

Part III: Cart with Cork Base

III.1. Make copies of any graphs, tables, or other elements of Capstone that you wish to include in your lab report before proceeding.

III.2. Repeat the procedure in Part I for the cart with the cork base. Record your percent difference below:

$$\frac{|mean\ coefficient - slope|}{\left|\frac{mean\ coefficient + slope}{2}\right|} \times 100\% = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \%$$

Part IV: Summary Table

Cart-Mass System	μ_s	μ_k
Felt + 0.250 kg		
Felt + 0.500 kg		
Felt + 0.750 kg		
Felt + 1.00 kg		
Plastic + 0.250 kg		
Plastic + 0.500 kg		
Plastic + 0.750 kg		

Cart-Mass System	μ_s	μ_k
Plastic + 1.00 kg		
Cork + 0.250 kg		
Cork + 0.500 kg		
Cork + 0.750 kg		
Cork + 1.00 kg		

Analysis

Note: In this section of the lab report, please include the summary table; you should also include the static vs normal and kinetic vs normal graphs for each type of cart. Please don't include the graphs without commenting on what they tell you. If you have nothing to say about a particular graph, it doesn't belong in your report.

1. Is one of the coefficients of friction always larger than the other? Why or why not?

2. Did the material on the base of the carts have any impact on the differences between the two coefficients of friction? Why or why not?

3. Did the masses loaded in the carts change the magnitudes of the forces of friction? In what way did the magnitudes change with mass? Did they change the coefficients of friction? Why or why not? (Hint: Forces of friction are due to the contact of materials; even though mass is used in our calculations, keep in mind the relationships $\vec{F}_s = \mu_s \vec{F}_N$ and $\vec{F}_k = \mu_k \vec{F}_N$ as you answer this question. Mass directly affects the magnitude of $\vec{F}_N$; so larger masses *will* increase the magnitudes of the frictional **forces** $\vec{F}_s$ and $\vec{F}_k$; but do larger masses change the **coefficients**?)

Discussion

Please use paragraphs to conclude your experiment. The conclusion needs to include but is not limited to:

1. What did you learn about the relationship(s) between static and kinetic friction and normal force over a surface? Consider the following questions as you attempt to answer this one:

   a. If you pull the cart while it is sitting on the board and the cart does not move, what forces are acting on or opposing the cart? How do the magnitudes of the acting forces compare to each other?

   b. What must happen to the forces acting on the cart sitting on the board in order for the cart to move? Where does this phenomenon occur in Figure 3?

   c. What is occuring in the flat section of Figure 3 with respect to these forces? In other words, which force has greater magnitude?

2. What must be true about the force exerted on an object placed on a surface in order for a force to pull or push an object along that surface and maintain a constant velocity? (Hint: One frictional force must be exceeded to set the object in motion; the other needs to be exceeded in order to *keep* the object in motion.)

3. Which did you expect to have the *smallest* value for μ_s—the felt, plastic, or cork based cart moving along the friction board? Why? Did your experimental results support your expectation?

4. Error analysis: You should have already computed the *percent differences* between graphically observed values of the coefficients of friction and values obtained from models that are derived from the experimental data. What types of errors (other than personal errors) may have contributed to the *percent differences* you encountered?